HW Chapter 9 Answers

1. What are the two types of hypotheses used in a hypothesis test? How are they related?

Ho: Null Hypotheses – A statement about a population parameter that is assumed to be true for the purposes of testing

Ha: Alternative Hypothesis - A statement about a population parameter that is assumed to be true is the Null Hypothesis is rejected during testing.

These two Hypotheses are complements of each other.

2. Describe the two types of error possible in a hypothesis test decision.
   Type I error: Rejecting a true Ho
   Type II error: Failing to reject a false Ho

True or False?
In Exercises 3-8, determine whether each statement is true or false. If it is false, rewrite it as a true statement.

3. 17. In a hypothesis test, you assume the alternative hypothesis is true. **False, you assume the Null Hypothesis is true.**

4. 18. A statistical hypothesis is a statement about a sample. **False, it is a statement about a population parameter.**

5. 19. If you decide to reject the null hypothesis, you can support the alternative hypothesis. **True**

6. 20. The level of significance is the maximum probability you allow for rejecting a null hypothesis when it is actually true. **True**

7. 21. A large P-value in a test will favor a rejection of the null hypothesis. **False, a small p-value supports rejecting the null hypothesis.**

8. 22. If you want to support a claim, write it as your null hypothesis. **False, to support a claim write it as the alternative hypothesis.**

Stating Hypotheses
In Exercises 9-15, use the given statement to represent a
claim. Write its complement and state which is Ho and which is *Ha.*

9. 23. Ha: $p > .65$ **Ho: $p \leq 65$**

10. 24. Ho: $\mu \leq 128$ **Ha: $\mu > 128$**

11. 25. Ha: $\sigma^2 \neq 5$ **Ho: $\sigma^2 = 5$**

12. 26. Ho: $\mu = 1.2$ **Ha: $\mu \neq 1.2$**

13. 27. Ho: $p \geq 0.45$ **Ha: $p < 0.45$**

14. 28. Ha: $\sigma < 0.21$ **Ho: $\sigma \geq 0.21$**

Think about the context of the claim. *Determine* whether you want to support or reject the claim.

a. State the null and alternative hypotheses in words.
b. Write the null and alternative hypotheses in appropriate symbols
c. Describe in words Type I error (the consequence of rejecting a true null hypothesis.)
d. Describe in words Type II error (the consequence of failing to reject a false null hypothesis.)

15. You represent a chemical company that is being sued for paint damage to automobiles. You want to support the claim that the mean repair cost per automobile is not $650. How would you write the null and alternative hypotheses?

    Ho: $\mu=650$ (cost is $650)   Ha: $\mu\neq650$ (Cost is not $650)
    Type I Error – Claim cost is not $650, when it actually is $650
    Type II Error – Cost is not $650, but fail to reject the claim that is $650.

16. You are on a research team that is investigating the mean temperature of adult humans. The commonly accepted claim is that the mean temperature is about 98.6°F. You want to show that this claim is false. How would you write the null and alternative hypotheses?=

    Ho: $\mu=98.6$ (Normal Temp is 98.6F)   Ha: $\mu\neq98.6$ (Normal Temp is not 98.6F)
    Type I Error – Claim normal temperature is not 98.6F, when it actually is 98.6F
    Type II Error – Normal Temperature is not 98.6F, but fail to detect that.

17. A light bulb manufacturer claims that the mean life of a certain type of light bulb is at least 750 hours. You are skeptical of this claim and want to refute it.

    Ho: $\mu\geq750$ (Bulbs last at least 750 hours) Ha: $\mu<750$ (Bulbs last less than 750 hours)
    Type I Error – Incorrectly claim light bulbs last less than 750 hours
    Type II Error – Fail to detect that light bulbs last less than 750 hours

18. As stated by a company's shipping department, the number of shipping errors per million shipments has a standard deviation that is less than 3. Can you support this claim?

    Ho: $\sigma\geq3$ (Standard Deviation of shipping errors is at 3) Ha: $\sigma<3$ (Standard Deviation of errors is under 3)
    Type I Error – Incorrectly claim Std Dev of Shipping errors is under 3
    Type II Error – Fail to detect that Std Dev of errors is under 3

19. A research organization reports that 33% of the residents in Ann Arbor, Michigan are college students. You want to reject this claim.

    Ho: $p=0.33$ (33% of residents are college students)   Ha: $p\neq0.33$ (It's not true 33% of residents are students)
    Type I Error – Incorrectly claim percentage of residents who are college students is not33%
    Type II Error – Fail to detect that residents who are college students is not33%

20. The results of a recent study show that the proportion of people in the western United States who use seat belts when riding in a car or truck is under 84%. You want to support this claim.

    Ho: $p\geq0.84$ (At least 84% of west people use seat belts)   Ha: $p<0.84$ (Less than 84% use seat belts)
    Type I Error – Incorrectly claim less than 84% of people in west use seat belts
    Type II Error – Fail to detect that less than 84% of people in west use seat belts

21. In your work for a national health organization, you are asked to monitor the amount of sodium in a certain brand of cereal. You find that a random sample of 82 cereal servings has a mean sodium content of 232 milligrams with a standard deviation of 10 milligrams. At $\alpha = 0.01$, can you conclude that the mean sodium content per serving of cereal is more than 230 milligrams?

(a) (DESIGN) State your Hypothesis

Ho: $\mu \le 230$

Ha: $\mu > 230$

(b) (DESIGN) State Significance Level of the test and explain what it means,

α=.01, which represents the maximum design probability of Type I error, which would be claiming the mean sodium per serving of cereal was over 230 mg, when the actual mean sodium was not over 230 mg.

(c) (DESIGN) Determine the statistical model (test statistic)

Test of mean vs. Hypothesized Value, population standard deviation unknown.

$$t \equiv \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \quad df = 81$$

Because the degrees of freedom is over 30, the t is approximately Standard Normal (Z)

This is a one-tail test, so α will be only in the upper tail.

(d) (DESIGN) Determine decision rule (pvalue method)

Reject Ho if p-value < α (.01)

(e) (DATA) Conduct the test and **circle** your decision

Z ≈ t = (232-230)/(10/sqrt(82)) = 1.81

p-value =P(Z>1.81) = 0.0351 > α

Fail to Reject Ho

(f) (CONCLUSION) State your overall conclusion in language that is clear, relates to the original problem and is consistent with your decision.

Insufficient data to conclude that mean cereal sodium is over 230 mg.

22. A tourist agency in Florida claims the mean daily cost of meals and lodging for a family of four traveling in Florida is $284. You work for a consumer protection advocate and want to test this claim. In a random sample of 50 families of four traveling in Florida, the mean daily cost of meals and lodging is $292 and the standard deviation is $25. At $\alpha = 0.05$, do you have enough evidence to reject the agency's claim?

(a) (DESIGN) State your Hypothesis

Ho: μ = 284
Ha: μ ≠ 284

(b) (DESIGN) State Significance Level of the test and explain what it means.

α=.05, which represents the maximum design probability of Type I error, which would be claiming the mean cost of lodging and meals is not $284, when it is.

(d) (DESIGN) Determine the statistical model (test statistic)

Test of mean vs. Hypothesized Value, population standard deviation unknown.

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \quad df = 49$$

Because the degrees of freedom is over 30, the t is approximately Standard Normal (Z)

This is a two-tail test, so α will need to be split into 2 parts.

(d) (DESIGN) Determine decision rule (critical value method)

Reject Ho if t >1.96 or t < -1.96

(e) (DATA) Conduct the test and **circle** your decision

t = (292-284)/(25/sqrt(50)) = 2.26

Reject Ho

(f) (CONCLUSION) State your overall conclusion i language that is clear, relates to the original problem and is consistent with your decision.

Cost for lodging and meals is not $284. The cost is actually more than that.

23. An environmentalist estimates that the mean waste recycled by adults in the United States is more than 1 pound per person per day. You want to test this claim. You find that the mean waste recycled per person per day for a random sample of 12 adults in the United States is 1.2 pounds and the standard deviation is 0.3 pound. At a = 0.05, can you support the claim?

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(c) (DESIGN) State your Hypothesis

Ho: μ ≤ 1
Ha: μ > 1

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(d) (DESIGN) Determine decision rule (critical value method)

Reject Ho if t >1.796

(e) (DATA) Conduct the test and **circle** your decision

t = (1.2-1.0)/(0.3/sqrt(12)) = 2.31

Reject Ho

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(d) (DESIGN) State Significance Level of the test and explain what it means.

α=.05, which represents the maximum design probability of Type I error, which would be claiming the environmentalist was correct when saying more than mean waste recycled is more than one pound per person per day, when in fact that is not true.

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(e) (DESIGN) Determine the statistical model (test statistic)

Test of mean vs. Hypothesized Value, population standard deviation unknown.

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \quad df = 11$$

Due to the small sample size, we must assume the data is approximately normal (or at least not heavily skewed) for the central limit theorem to apply.

This is a one-tail test, so α will be only in the upper tail.

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(f) (CONCLUSION) State your overall conclusion in language that is clear, relates to the original problem and is consistent with your decision.

The environmentalist is correct. Mean waste recycled exceeds more than one pound per person per day.

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24. A government association claims that 44% of adults in the United States do volunteer work. You work for a volunteer organization and are asked to test this claim. You find that in a random sample of 1165 adults, 556 do volunteer work. At $\alpha = 0.05$, do you have enough evidence to reject the association's claim?

(a) (DESIGN) State your Hypothesis

Ho: p = 0.44
Ha: p ≠ 0.44

(b) (DESIGN) State Significance Level of the test and explain what it means,

α=.05, which represents the maximum design probability of Type I error, which would be claiming the government association was incorrect in its claim that 44% of US adults do volunteer work, when in fact the association's claim was correct.

(c) (DESIGN) Determine the statistical model (test statistic)

Test of Proportion vs. Hypothesized Value

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{(p_0)(1-p_0)}{n}}}$$

Since np(1-p)>5, we can use the normal approximation.

This is a two-tail test, so α and the p-value will be split into the upper and lower tails.

(d) (DESIGN) Determine decision rule (pvalue method)

Reject Ho if p-value < α (.05)

(e) (DATA) Conduct the test and **circle** your decision

$$\hat{p} = \frac{556}{1165} = 0.477$$

$$Z = \frac{0.477 - 0.44}{\sqrt{\frac{(0.44)(1-0.44)}{1165}}} = 2.54$$

$$P(Z < -2.54) = .0055$$

$$P(Z > 2.54) = .0055$$

p-value=2(0.0052)=0.0110 < α (.05)

Reject Ho

(f) (CONCLUSION) State your overall conclusion in language that is clear, relates to the original problem and is consistent with your decision.

The government association is wrong; the percentage of US adults who do volunteer work is not 44%. In fact, it is more than 44%

25. Additional Problems: The geyser Old Faithful in Yellowstone National Park is claimed to erupt for on average for about three minutes. Thirty-six observations of eruptions of the Old Faithful were recorded (time in minutes)

1.8	1.98	2.37	3.78	4.3	4.53
1.82	2.03	2.82	3.83	4.3	4.55
1.88	2.05	3.13	3.87	4.43	4.6
1.9	2.13	3.27	3.88	4.43	4.6
1.92	2.3	3.65	4.1	4.47	4.63
1.93	2.35	3.7	4.27	4.47	6.13

Sample mean = 3.394 minutes. Sample standard deviation = 1.168 minutes

Test the hypothesis that the mean length of time for an eruption is 3 minutes.

1. General Question
   a. Why do you think this test is being conducted?
      We are trying to test the claim that Old Faithful erupts for three minutes.
2. Design
   a. State the null and alternative hypotheses
      Ho: $\mu = 3$ Ho: $\mu \neq 3$
   b. What is the appropriate test statistic/model?
      t -Test of mean vs. Hypothesized Value, population standard deviation unknown.

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \quad df = 35$$

   c. What is significance level of the test?
      Not given, so I will choose α = 5%
   d. What is the decision rule?
      Critical value method: Reject Ho if t<-1.96 ot t>1.96
3. Conduct the test
   a. Are there any unusual observations that question the integrity of the data or the assumptions of the model? (additional problem only)
      The value 6.13 appears to be an outlier and increases the standard deviation.
   b. Is the decision to reject or fail to reject Ho?

$$t = \frac{3.394 - 3}{1.168/\sqrt{36}} = 2.02 \quad \textbf{Reject Ho (barely)}$$

4. Conclusions - State a one paragraph conclusion that is consistent with the decision using language that is clearly understood in the context of the problem. Address any potential problems with the sampling methods and address any further research you would conduct.

The mean length of time for eruptions of Old Faithful is not three minutes, it appears to be longer. This conclusion is based on 36 measurements taken of Old Faithful.

One measurement of 6.13 seems highly unusual and does not seem to fit the data. I would review the data to make sure this result was recorded correctly.

26. Define the following terms – **All answers are in glossary of my Online text.**

27. A study claims more than 60% of students text-message frequently. In a poll of 1000 students, 660 students said they text message frequently. Can you support the study's claim? Conduct the test with α = 1%

Ho: p ≤ 0.60 Ha: p > 0.60

α = 1% **Model:** $Z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{(p_0)(1-p_0)}{n}}}$

Reject Ho if p-value <.01

$$\hat{p} = \frac{660}{1000} = 0.66$$

$$Z = \frac{0.66 - 0.60}{\sqrt{\dfrac{(0.60)(1-0.60)}{100}}} = 3.87$$

$$P(Z > 3.87) \approx .0000$$

Reject Ho

The study is correct. More than 60% of students text-message frequently.

28. 15 I-pod users were asked how many songs were on their I-pod. Here are the summary statistics of that study:

$$\overline{X} = 650 \qquad s = 200$$

a. Can you support the claim that the number of songs on a user's I-pod is different from 500? Conduct the test with α= 5% .

Ho: μ = 500 Ha: $\mu \neq$ 500
Test of mean vs. Hypothesized Value, population standard deviation unknown.

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \quad df = 14$$

Due to the small sample size, we must assume the data is approximately normal (or at least not heavily skewed) for the central limit theorem to apply.
Reject Ho if t >2.145 or t<-2.145 (Two tailed test)

t = (650-500)/(200/sqrt(15)) = 2.90 → Reject Ho

The mean number of songs on a user's Ipod is not 500. It is more.

b. Can you support the claim that the population standard deviation is under 300? Conduct the test with α= 5% .

Ho: $\sigma \geq$ 300 Ha: σ < 300
Chi square test of standard deviation vs. Hypothesized Value

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad df = 14 \qquad \alpha = .05$$

Reject Ho if χ^2 <6.571 (lower tailed test)

1. $\chi^2 = \frac{(14)200^2}{300^2} = 6.22$ → **Reject Ho**

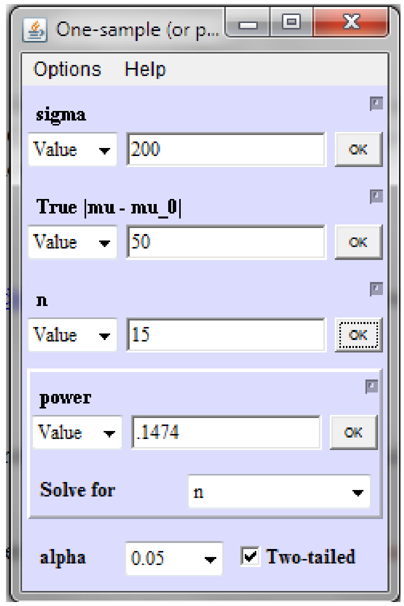
The standard deviation is under 300.

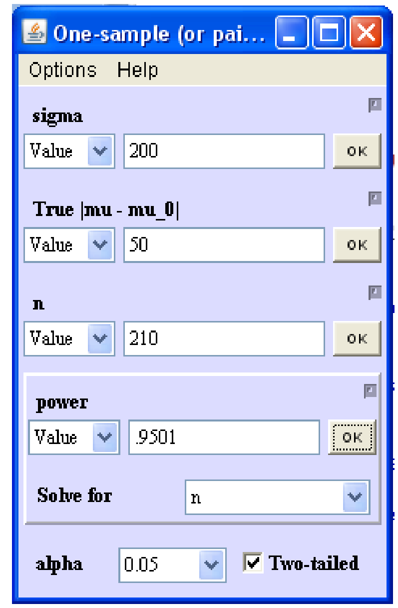
29. Consider the design procedure in the test you conducted in Question 8a. Suppose you wanted to conduct a Power analysis if the population mean under Ha was actually 550. Use the online Power calculator to answer the following questions.

a. Determine the Power of the test.
power =.1474

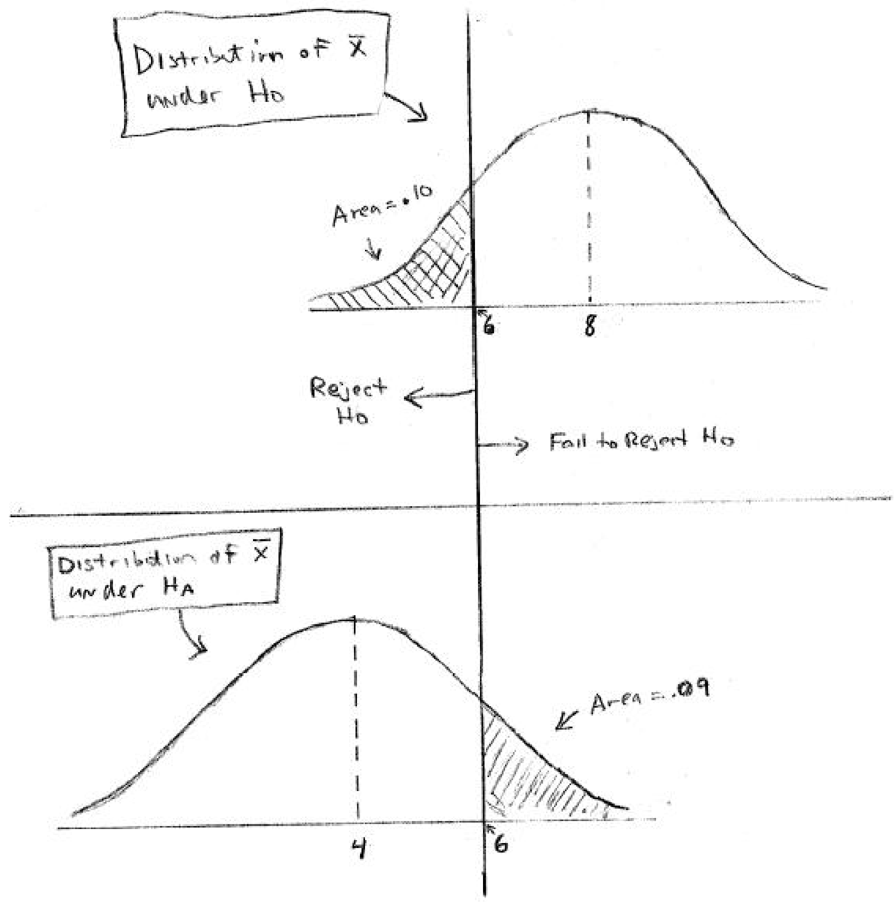
b. Determine Beta.
β=1-power = .8603

c. Determine the sample size needed if you wanted to conduct the test in Question 8a with 95% power
n=210





30. The drawing shown diagrams a hypothesis test for population mean design under the Null Hypothesis (top drawing) and a specific Alternative Hypothesis (bottom drawing). The sample size for the test is 200.



a. State the Null and Alternative Hypotheses
Ho: μ ≥ 8 Ha: μ < 8

b. What are the values of μ_0 and μ_a in this problem?
μ_0 = 8 Ha: μ_α = 4

c. What is the significance level of the test? **α=.10**

d. What is the Power of the test when the population mean = 4?
Power = .91

e. Determine the probability associated with Type I error.
α=.10

f. Determine the probability associated with Type II error.
β=.09

g. Under the Null Hypothesis, what is the probability the sample mean will be over 6?
1−α=.90

h. If the significance level was set at 5%, would the power increase, decrease or stay the same?
Power would decrease

i. If the test was conducted, and the p-value was .085, would the decision be Reject or Fail to Reject the Null Hypothesis? **Reject Ho because .085<.10**

j. If the sample size was changed to 100, would the shaded on area on the bottom (Ha) graph increase, decrease or stay the same? **Increase: Beta would increase when sample size decreases.**