Lesson 4.2.1
Exponential Models & Logarithms

SACWAY
BRAINSTORMING ALGEBRA & STATISTICS

STUDENT NAME ______________________________ DATE ______________

INTRODUCTION

The Cell Cycle

The *cell cycle* is the process by which living cells replicate. The cell cycle is a relatively constant time period during which cells reproduce. For most mammals, the average cell cycle is very nearly one day. At the end of this cycle, a single cell experiences *cytokinesis*, where it divides into two cells. At the end of another cycle, each of these cells divides, and then there are *four* cells. In the discussion that follows, we assume that a population of cells grows from a single cell (beginning on day 0). This population will have adequate space and nutritional resources to allow continuous growth, and that no cells cease replication through a given time period. We begin by choosing a cell which has a cycle of 1 day.

1. Use a calculator with an exponent button ([x^y] or [^]) to complete the table below.

t **(Days)**	***A*** **(Number of cells)**	**Rewrite *A* as a power of 2.**
0	1	$1 = 2^0$
1	2	$2 = 2^1$
2		
3		
4		
5		
6		
7		
8		
⋮	⋮	⋮
15		
19		
⋮	⋮	⋮
t	→	

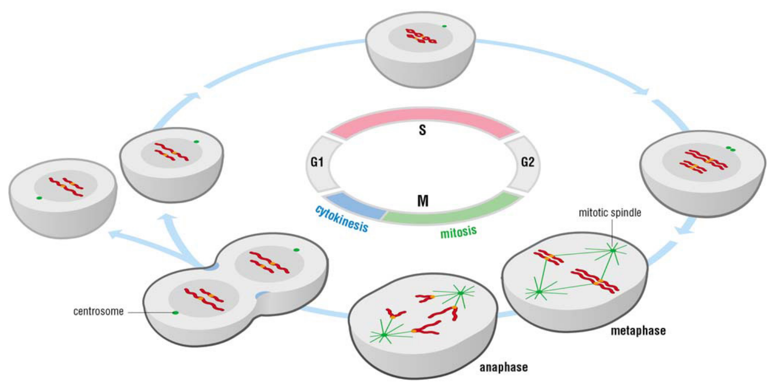

Figure 1: The cell cycle. From The Cell Cycle: Principles of Control by David O. Morgan. ©1999 – 2007, New Science Press. Used by permission.

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2. Write an *exponential* equation that gives the number of cells, A, after t days.

$$A =$$

3. Use trial and error on your calculator to estimate the time when there will be 2048 cells. In doing so, you are solving the equation $2^t = 2048$.

Keep in mind that the equation which you have created is only a model, a *function* which *estimates* the number of cells after t days. It will not be perfectly correct, because all cell cannot be expected to complete their cycles in exactly one day, nor all at the same instant. In fact, over time, we can expect the cell divisions to occur at any time of the day, as their cycles lose synchronization.

In fact, it is reasonable to expect that, while the experiment proceeds, any positive (whole) number of cells could be observed (up to some maximum number). When $t = 7.65$ days, we can expect around

$$A = 2^{7.65} \approx 200 \text{ cells.}$$

4. Use trial and error with the exponential function on your calculator, to find a fractional number of days (not a whole number) when there are around 300 cells. In doing so, you are solving the equation $2^t = 300$.

$$t =$$

5. Use trial and error with the exponential function on your calculator, to find a fractional number of days (not a whole number) when there are around 950 cells. In doing so, you are solving the equation $2^t = 950$.

$$t =$$

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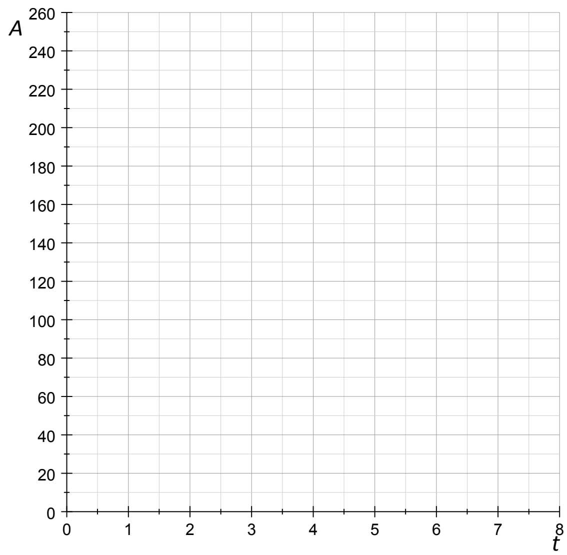
6. Graph your exponential function, $A = 2^t$, below. Use the points computed in the table from Question 1.



7. Use your graph to estimate the (fractional) number of days when there will be 100 cells.

NEXT STEPS

Finding the exponent that causes an exponential function to yield a particular value can be challenging. This is particularly true when the exponent is *not* a whole number.

Scientific calculators have a tool which can give the exponent of a base that will yield a particular value. That tool is called a *logarithm*. In problem 7 above you used a graph to determine the power of 2 which will yield 100. There are multiple ways to use a logarithm to give this answer. One way is to use a *natural logarithm* (denoted *ln*) function as follows (we will talk more about this later):

$$\frac{\ln(100)}{\ln(2)}.$$

The 100 in the expression is the resulting number of cells we were trying to achieve, the 2 is the base of the exponential function.

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TRY THESE

8. Use your calculator to evaluate this expression.

$$\frac{\ln(100)}{\ln(2)} =$$

9. How does your answer compare to the value determined graphically in problem 7?

NEXT STEPS

What is a Logarithm?

A logarithm is used to give the value of an exponent in an exponential function. The simplest form of an exponential function looks like this:

$$b^x.$$

In this function, the value, b, is the *base* of the exponential, and x is the *exponent*. The logarithm which can tell us the exponent in a base b exponential is a *base b logarithm*.

Logarithms appear in various forms, but a *base b logarithmic function* looks like this:

$$\log_b(x)$$

This logarithm gives the power to which b must be raised to yield x. That is,

$$\log_b(x) = \text{the power to which } b \text{ must be raised to yield } x.$$

For example, to evaluate $\log_2(32)$, we ask "2 to the *what* yields 32?" Since $2^5 = 32$, we know the answer is 5.

$$\log_2(32) = 5$$

Evaluating the logarithm, $\log_2(32)$ is equivalent to solving the equation $2^t = 32$.

In summary, to evaluate $\log_b(x)$, we ask the equivalent question, "b to the *what* yields x?"

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10. Give questions and equations that are equivalent to the logarithms below. Finally, solve the equations.

Logarithm	Equivalent Question	Equation	Solution
$\log_2(8)$	2 to the *what* yields 8?	$2^t = 8$	3
$\log_3(81)$			
$\log_5(5)$			
$\log_7(49)$			

11. *Exponential equations* have a variable in an exponent. We often solve them using logarithms. In the table below, convert the equation to a question, then a logarithm, and solve it.

Equation	Equivalent Question	Logarithm	Solution
$3^t = 9$	3 to the *what* yields 9?	$\log_3(9)$	2
$10^t = 1000$			
$20^t = 20$			
$3^t = 243$			

NEXT STEPS

There are many types of logarithms, but your calculator only has a few. To evaluate logarithms, we use a mathematical trick to convert them to the *natural* logarithm like this:

$$\log_b(x) = \frac{\ln(x)}{\ln(b)}.$$

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12. Use your calculator to evaluate the logarithms below.

Equation	Logarithm	Convert to natural logarithm	Evaluate
$4^t = 9$	$\log_4(9)$	$\frac{\ln(9)}{\ln(4)}$	1.585
$7^x = 135$			
$9^x = 1000$			
$5^x = 252$			

SUMMARY

Let's summarize what we know about logarithms.

I. To evaluate the logarithm $\log_b(x)$ ask "*b* to the what yields *x*?"
II. The equation $b^t = x$ asks the same question: "*b* to the what yields *x*?"
It is answered by the logarithm: $t = \log_b(x)$.
III. To evaluate a logarithm on a calculator, we use a trick to convert to the natural logarithm (ln).

$$\log_b(x) = \frac{\ln(x)}{\ln(b)}$$

TRY THESE

Refer back to the exponential model which you used to quantify the number of cells in the population described in Problem 2. Use logarithms to determine when there will be approximately 3500 cells.

13. Rewrite the problem as an equation, using the model from 2, where the number of cells is $A = 3500$. Next, rewrite the equation as a base 2 logarithm, then evaluate on your calculator using the natural logarithm function.

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TAKE IT HOME

Before answering the questions below, review the following rule of exponents:

$b^0 = 1$ for any number a, $a \neq 0$.

For example: $2^0 = 1$, and also $15^0 = 1$.

1 Rewrite each of the following logarithmic expressions as an exponential equation and then solve. (See #10 above).

a. $\log_2(4) =$

b. $\log_3(27) =$

c. $\log_6(6) =$

d. $\log_2(32) =$

e. $\log_5(125) =$

f. $\log_3(3) =$

g. $\log_5(1) =$

2 Rewrite each of the following exponential equations as a logarithmic expression and then solve for t. (See Question 11 in the classroom activity above.)

a. $3^t = 81$

b. $2^t = 16$

c. $5^t = 625$

d. $2^t = 128$

e. $3^t = 1$

f. $4^t = 64$

g. $5^t = 5$

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3 Write the following exponential equations as a logarithmic expression and use your calculator to evaluate your answer. (See Question 12 in the classroom activity above.)

a. $2^x = 7$

b. $5^n = 100$

c. $3^z = 65$

d. $2^t = 70$

e. $3^t = 300$

f. $4^w = 190$

g. $9^r = 250$

4 The exponential equation below is used to model the number of people who live in Washmont City, where *A* represents the number of people who live in Washmont after *t* years.

$$A = 3^t$$

a. Estimate the number of people who will be living in Washmont after 10 years? (What is the value of *A* when $t = 10$?) Round your answer to the nearest whole person.

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b. Estimate the number of years required for the population of Washmont to reach 2000 people? (What is the value of t when $A = 2000$?) Round your answer to the nearest hundredth.

5 The exponential equation below is used to model the population of bacteria in a culture, where A represents the number of bacteria present after t days.

$$A = 5.4^t$$

a. Estimate the number of bacteria that will be present after 4 days? (What is the value of A when $t = 4$?)

b. Estimate the number of days for 300 bacteria to be present? (What is the value of t when $A = 300$?) Round your answer to the nearest hundredth.

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