

Math 10 - Homework 4 MPS

1. High Fructose Corn Syrup (HFCS) is a sweetener in food products that is linked to obesity and type II diabetes. The mean annual consumption in the United States in 2008 of HFCS was 60 lbs with a standard deviation of 20 lbs. Assume the population follows a Normal Distribution.
- Find the probability a randomly selected American consumes more than 50 lbs of HFCS per year.
 $P(X > 50) = P(Z > (50 - 60)/20) = P(Z > -0.50) = 1 - .3085 = .6915$
 - Find the probability a randomly selected American consumes between 30 and 90 lbs of HFCS per year.
 $P(30 < X < 90) = P((30 - 60)/20 < Z < (90 - 60)/20) = P(-1.50 < Z < 1.50) = .9332 - .0668 = .8664$
 - Find the 80th percentile of annual consumption of HFCS.
 $Z_{80} = 0.84 \quad X_{80} = 60 + (0.84)(20) = 76.8 \text{ lbs. per year}$
 - In a sample of 40 Americans how many would you expect consume more than 50 pounds of HFCS per year.
 $P(X > 50) = .6915$ from part A. **Expected Value = $40(.6915) = 27.7$ or about 28 out of 40.**
 - Between what two numbers would you expect to contain 95% of Americans HFCS annual consumption?
 $(60 - 1.96(20), 60 + 1.96(20))$ or 20.8 to 99.2 lbs. per year
 - Find the quartiles and Interquartile range for this population.
 $Z_{25} = -0.67 \quad Z_{50} = 0 \quad Z_{75} = 0.67 \quad X_{25} = 60 - (0.67)(20) = 46.6 \text{ lbs} \quad X_{50} = 60 \text{ lbs} \quad X_{75} = 60 + (0.67)(20) = 73.4 \text{ lbs}$
IQR = $73.4 - 46.6 = 26.8$ lbs per year
 - A teenager who loves soda consumes 105 lbs of HFCS per year. Is this result unusual? Use probability to justify your answer.
 $P(X > 105) = P(Z > (105 - 60)/20) = P(Z > 2.25) = 1 - .9878 = .0122$ **Unusual result**
 - In a sample of 16 Americans, what is the probability that the **sample mean** will exceed 57 pounds of HFCS per year?
 $P(\bar{X} > 57) = P(Z > (57 - 60)/(20/\sqrt{16})) = P(Z > -0.60) = 1 - .2743 = .7257$
 - In a sample of 16 Americans, what is the probability that the **sample mean** will be between 50 and 70 pounds of HFCS per year.
 $P(50 < \bar{X} < 70) = P((50 - 60)/(20/\sqrt{16}) < Z < (70 - 60)/(20/\sqrt{16}))$
 $= P(-2.00 < Z < 2.00) = .9772 - 0.0228 = 0.9544$
 - In a sample of 16 Americans, between what two values would you expect to see 95% of the **sample means**?
 $60 - 1.96 * 20/\sqrt{16}$, $60 + 1.96 * 20/\sqrt{16}$
Between 50.2 and 69.8 lbs

2. A normally distributed population of package weights has a *mean* of 63.5 g and a *standard deviation* of 12.2 g.
- What percentage of this population weighs 66 g or more?
 $P(X > 66) = P(Z > (66 - 63.5) / 12.2) = P(Z > 0.20) = 1 - .5793 = .4207$
 - What percentage of this population weighs 41 g or less?
 $P(X < 41) = P(Z < (41 - 63.5) / 12.2) = P(Z < -1.84) = .0329$
 - What percentage of this population weighs between 41 g and 66 g?
 $P(41 < X < 66) = P((41 - 63.5) / 12.2 < Z < (66 - 63.5) / 12.2) = P(-1.84 < Z < 0.20) = .5793 - .0329 = .5464$
 - Find the 60th percentile for distribution of weights.
 $Z_{60} = 0.25 \quad X_{60} = 63.5 + (0.25)(12.2) = 66.55 \text{ g}$
 - Find the three quartiles and the interquartile range.
 $Z_{25} = -0.67 \quad Z_{50} = 0 \quad Z_{75} = 0.67 \quad X_{25} = 63.5 - (0.67)(12.2) = 55.3 \text{ g} \quad X_{50} = 63.5 \quad X_{75} = 63.5 + (0.67)(12.2) = 71.7 \text{ g}$
 $IQR = 71.7 - 55.3 = 16.4 \text{ g}$
 - If you sample 49 packages, find the probability the sample mean is over 66 g. Compare this answer to part a.
 $P(\bar{X} > 66) = P(Z > (66 - 63.5) / [12.2 / \sqrt{49}]) = P(Z > 0.82) = 1 - .7939 = .2061$ lower since sdev is lower.
 - If you sample 49 packages, find the probability the sample mean is over 66 g. Compare this answer to part a.
 $P(\bar{X} > 66) = P(Z > (66 - 63.5) / [12.2 / \sqrt{49}]) = P(Z > 1.43) = 1 - .9236 = .0764$ lower since sdev is lower.
3. A pollster sampled 100 adults in California and asked a series of questions. The Central Limit Theorem for Proportions requires that $np > 10$ and $n(1-p) > 10$. Determine if these conditions are met for the following statements.
- 61% of Californians live in Southern California.
 $np = 61, n(1-p) = 39$ – both more than 10 - yes
 - 92% of Californians support Deferred Action for Childhood Arrivals (DACA)
 $np = 92, n(1-p) = 8$ – $n(1-p)$ less than 10 - no
 - 8% of Californians have a felony conviction.
 $np = 8, n(1-p) = 92$ – np less than 10 - no
4. 24% of Californians have visited Yosemite National Park. A pollster samples 1000 Californians.
- Determine the expected value and standard deviation of the sample proportion.
 $\text{Mean} = p = 0.24 \quad \text{std dev} = \sqrt{0.24(1-0.24)/1000} = 0.0135$
 - Determine that the condition for normality is satisfied.
 $np = 240, n(1-p) = 760$ – both more than 10 - yes
 - Determine the probability the sample proportion exceeds 0.40.
 $P(\hat{p} > 0.40) = P(Z > (0.40 - 0.24) / 0.0135) = P(Z > 11.85) = 0$