Math 10 - Homework 4 MPS

1. High Fructose Corn Syrup (HFCS) is a sweetener in food products that is linked to obesity and type II diabetes. The mean annual consumption in the United States in 2008 of HFCS was 60 lbs with a standard deviation of 20 lbs. Assume the population follows a Normal Distribution.

   a. Find the probability a randomly selected American consumes more than 50 lbs of HFCS per year.
   $P(X>50) = P(Z>(50-60)/20)=P(Z>-0.50)=1-.3085=.6915$

   b. Find the probability a randomly selected American consumes between 30 and 90 lbs of HFCS per year.
   $P(30<X<90) = P((30-60)/20)<Z<(90-60)/20)=P(-1.50<Z<1.50)=.9332-.0668= .8664$

   c. Find the 80th percentile of annual consumption of HFCS.
   $Z_{80}=0.84$ $X_{80}=60+(0.84)(20)=76.8$ lbs. per year

   d. In a sample of 40 Americans how many would you expect consume more than 50 pounds of HFCS per year.
   $P(X>50) = .6915$ from part A. Expected Value = 40(.6915) = 27.7 or about 28 out of 40.

   e. Between what two numbers would you expect to contain 95% of Americans HFCS annual consumption?
   (60-1.96(20), 60+1.96(20)) or 20.8 to 99.2 lbs. per year

   f. Find the quartiles and Interquartile range for this population.
   $Z_{25}=-0.67$ $Z_{50}=0$ $Z_{75}=0.67$ $X_{25}=60-(0.67)(20)=46.6$ lbs $X_{50}=60$ lbs $X_{75}=60+(0.67)(20)=73.4$ lbs
   IQR=73.4-46.6= 26.8 lbs per year

   g. A teenager who loves soda consumes 105 lbs of HFCS per year. Is this result unusual? Use probability to justify your answer.
   $P(X>105) = P(Z>(105-60)/20)=P(Z>2.25)=1-.9878=.0122$ Unusual result

   h. In a sample of 16 Americans, what is the probability that the **sample mean** will exceed 57 pounds of HFCS per year?
   $P(XBAR>57) = P(Z>(57-60)/(20/sqrt(16))=P(Z>-0.60)=1-.2743 = .7257$

   i. In a sample of 16 Americans, what is the probability that the **sample mean** will be between 50 and 70 pounds of HFCS per year.
   $P(50<XBAR<70) = P((50-60)/(20/sqrt(16))<Z<(70-60)/(20/sqrt(16))$
   $=P(-2.00<Z<4.00)=1-.2743 = 1 – 0.9772 = .0228$

   j. In a sample of 16 Americans, between what two values would you expect to see 95% of the **sample means**?
   60 – 1.96*20/sqrt(16) , 60 + 1.96*20/sqrt(16)
   Between 40.2 and 79.8 lbs

2. A normally distributed population of package weights has a *mean* of 63.5 g and a *standard deviation* of 12.2 g.

   a. What percentage of this population weighs 66 g or more?
   P(X>66) = P(Z>(66-63.5)/12.2)=P(Z>0.20)=1-.5793=.4207

   b. What percentage of this population weighs 41 g or less?
   P(X<41) = P(Z<(41-63.5)/12.2)=P(Z<-1.84)=.0329

   c. What percentage of this population weighs between 41 g and 66 g?
   P(41<X<66) = P((41-63.5)/12.2)<Z<(66-63.5)/12.2)=P(-1.84<Z<0.20)=.5793-.0329=.5464

   d. Find the 60^{th} percentile for distribution of weights.
   Z_{60}=0.25 X_{60}=63.5+(0.25)(12.2)=66.55 g

   e. Find the three quartiles and the interquartile range.
   Z_{25}=-0.67 Z_{50}=0 Z_{75}=0.67 X_{25}=63.5-(0.67)(12.2)=55.3g X_{50}=63.5 X_{75}=63.5+(0.67)(12.2)=71.7g
   IQR=71.7-55.3=16.4 g

   f. If you sample 49 packages, find the probability the sample mean is over 66 g. Compare this answer to part a.
   P(Xbar>66) = P(Z>(66-63.5)/[12.2/sqrt(16)]=P(Z>0.82)=1-.7939=.2061 lower since sdev is lower.

   g. If you sample 49 packages, find the probability the sample mean is over 66 g. Compare this answer to part a.
   P(Xbar>66) = P(Z>(66-63.5)/[12.2/sqrt(49)]=P(Z>1.43)=1-.9236=.0764 lower since sdev is lower.

3. A pollster sampled 100 adults in California and asked a series of questions. The Central Limit Theorem for Proportions requires that np > 10 and n(1-p) > 10. Determine if these conditions are met for the following statements.

   a. 61% of Californians live in Southern California.
   np=61, n(1-p)=39 – both more than 10 - yes

   b. 92% of Californians support Deferred Action for Childhood Arrivals (DACA)
   np=92, n(1-p)=8 – n(1-p) less than 10 - no

   c. 8% of Californians have a felony conviction.
   np=8, n(1-p)=92 – np less than 10 - no

4. 24% of Californians have visited Yosemite National Park. A pollster samples 1000 Californians.

   a. Determine the expected value and standard deviation of the sample proportion.
   Mean = p = 0.24 std dev = sqrt(0.24(1-0.24)/1000) = 0.0135

   b. Determine that the condition for normality is satisfied.
   np=240, n(1-p)=760 – both more than 10 - yes

   c. Determine the probability the sample proportion exceeds 0.40.
   P(p-hat>0.40) = P(Z>(0.40 – 0.24)/0.0135) = P(Z>11.85) = 0