Normal Distribution/Central Limit Theorem **Answers** (gw 11)

1. State the 3 important parts of the Central Limit Theorem for the sample mean $\overline{X}$

   Mean stays the same

   Standard deviation gets smaller in proportion to the square root of the sample size

   When n is sufficiently large, sample mean has approximately a **Normal Distribution**

2. The completion time (in minutes) for a student to complete a short quiz follows the continuous probability density function shown here, with some areas calculated. It is known that μ=5.3 minutes and σ = 2.4 minutes. 40 students take the quiz.

   a. Find the probability that the **mean** completion time for the students is under 5 minutes.

$$P(X<5)=P\left(Z<\frac{5-5.3}{2.4/\sqrt{40}}\right)=P(Z<-0.79)=0.2148$$

   b. Find the probability that the **mean** time for the class to finish the quiz is between 6 and 8 minutes.

$$P(6\leq X\leq 8)=P\left(\frac{6-5.3}{2.4/\sqrt{40}}\leq Z\leq\frac{8-5.3}{2.4/\sqrt{40}}\right)=P(1.84\leq Z\leq 7.12)=1-0.9671=0.0329$$

   c. The **mean** completion time for the class was 7.1 minutes. Is this result unusual? Explain.

$$Z=\frac{7-5.3}{2.4/\sqrt{40}}=4.48$$ Yes, this is a very unusual result

3. For women aged 18-24, systolic blood pressures (in mmHg) are normally distributed with μ=114.8 and σ=13.1.

   a. Find the probability a woman aged 18-24 has systolic blood pressure exceeding 120.

$$P(X>120)=P\left(Z>\frac{120-114.8}{13.1}\right)=P(Z>0.40)=1-.6554=.3446$$

   b. If 40 women are randomly selected, find the probability that their mean blood pressure exceeds 120.

$$P(\overline{X}>120)=P\left(Z>\frac{120-114.8}{13.1/\sqrt{40}}\right)=P(Z>2.51)=1-.9940=.0060$$

   c. If the pdf for systolic blood pressure did NOT follow a normal distribution, would your answer to part b change?

   Answer would stay the same. When n is sufficiently large, Sample mean has approximately a Normal Distribution regardless of the shape of the pdf of X.

4. It has been reported that 28% of all Californians have visited Yosemite National Park. A pollster samples 1000 Californians and determines that 240 of them have visited Yosemite National Park.

   a. Determine the value of the sample proportion, $\hat{p}$, of Californians who have visited Yosemite. Is the higher or lower than the reported population value?

      $\hat{p} = 240/1000 = 0.24$ This is lower than the reported population value

   b. Determine the expected value of the sample proportion.

      $\mu_{\hat{p}} = p = 0.28$

   c. Determine the standard deviation of the sample proportion.

      $$\sigma_{\hat{p}} = \sqrt{\frac{0.28(1-0.28)}{1000}} = 0.0142$$

   d. Determine that the condition for normality is satisfied.

      1000(.28)=280   1000(1-.28) = 720. Both values are greater than 10, so conditions for normality are met.

   e. Determine the probability the sample proportion exceeds 0.24.

      $$P(\hat{p} > 0.24) = P\left(Z > \frac{0.24-0.28}{0.0142}\right) = P(Z > 2.82) = 1 - 0.9976 = 0.0024$$

   f. Determine the probability the sample proportion is between 0.2 and 0.3.

      $$P(0.2 \le \hat{p} \le 0.3) = P\left(\frac{0.2-0.28}{0.142} \le Z \le \frac{0.3-0.28}{0.142}\right) = P(-5.63 \le Z \le 1.41) = 0.9207 - 0 = 0.9207$$