Math 10 - Exam 2 Topics

You will be tested on defining, calculating, interpreting and explaining topics from the following list (not all topics are necessarily on the test):

- **Confidence Intervals**
  - Level of Confidence
  - Margin of Error
    - Effect of Sample Size
    - Effect of Confidence Level
    - Effect of Standard Deviation
    - Effect of model (Z vs. t)
  - Confidence Interval for μ
    - Population Standard Deviation known (Z)
    - Population Standard Deviation unknown (t)
    - Determining Sample size for known Margin of Error
  - Confidence Interval for Proportion p
    - Determining Sample size for known Margin of Error
  - Confidence Interval for variance σ^2 and standard deviation σ

- **Hypothesis Testing**
  - Concepts Definitions
    - Null and Alternative Hypotheses
    - One and Two Tailed Tests
    - Type I and Type II errors
    - Level of Significance - α
    - Test Statistic – Model
    - Critical Value
    - p-value
    - Decision Rule
      - Critical Value Method
      - p-value Method
    - Power
    - Beta
    - Effect Size
    - Decision
    - Conclusion
  - One population Tests
    - **Large sample – Z-Test for μ vs. hypothesized value**
    - **Small Sample – t-test for μ**
    - **Z-test for proportion (p) vs. hypothesized value**
    - **χ^{2-} test for σ vs. hypothesized value**
  - Two Population Tests for comparing means
    - Independent Sampling
      - Large Sample (n_1,n_2>30) – **two population Z-test**
      - Small Sample, $\sigma_1^2 = \sigma_2^2$ – **pooled variance t-test**
      - Small Sample, $\sigma_1^2 \neq \sigma_2^2$ – **unequal variance t-test**
    - Dependent Sampling – **Matched pair t-test**
  - Compare 2 variances – **F test for comparing σ_1 and σ_2**

- You may bring 4 pages of **HANDWRITTEN** notes to the exam.
- Bring your probability tables, Picture ID, Pencil, Calculator and your notes to the exam–**no sharing** is allowed during the exam. No cell phone calculators.
- Cell Phones, iPods, PDAs, and other electronic devices must be **turned off** and **put away**.
- Manage your time so you can **attempt every question**.

Math 10 – Practice Exam 2

A January, 2006 CBS Poll showed that 55% of American adults "believe that America is ready to elect a woman president." The sample size for this poll was 1300.

a. Calculate the **margin of error** (round to 3 decimals) for a 95% confidence interval for the proportion of all American adults who "believe that America is ready to elect a woman president."

b. Read these proposed changes to the confidence interval and check one answer below.

A. Increase sample size to 2600

B. Decrease confidence level to 90%

☐ Both A and B increase the Margin of Error.
☐ A increases the margin of error and B decreases the Margin of Error.
☐ B increases the margin of error and A decreases the Margin of Error.
☐ Both A and B decrease the Margin of Error.

2. A commuter recorded the minutes spent driving home on 15 randomly selected days:

29	34	41
29	36	41
30	36	41
32	38	50
33	39	58

$\overline{X} = 37.8$ $s = 7.94$

a. Find a 99% confidence interval for the population mean.

b. If the sample size increased from 15 to 30, would the margin of error for the confidence interval (mark one answer):
☐ increase ☐ decrease ☐ stay the same ☐ not enough information to answer

c. If the confidence level decreased from 99% to 95%, would the margin of error for the confidence interval (mark one answer):
☐ increase ☐ decrease ☐ stay the same ☐ not enough information to answer

d. If the sample standard deviation changed from 7.94 to 9.05, would the margin of error for the confidence interval (mark one answer):
☐ increase ☐ decrease ☐ stay the same ☐ not enough information to answer

e. If **7.94** was the **population standard deviation** instead of the sample standard deviation, would the margin of error for the confidence interval (mark one answer):
☐ increase ☐ decrease ☐ stay the same ☐ not enough information to answer

3. The number of units taken by students at a community college is approximately normal. A sample of 20 is selected and the sample statistics are: $\overline{X} = 11.2\ units$ $s = 5.1\ units$.

- Find a for a 90% confidence interval for the population mean.
- If the confidence level increased from 90% to 95%, would the margin of error for the confidence interval would (mark one answer):
☐ **increase** ☐ **decrease** ☐ **stay the same** ☐ **not enough information to answer**
- If the student population was changed to include university students as well, the margin of error for the confidence interval would (mark one answer):
☐ **increase** ☐ **decrease** ☐ **stay the same** ☐ **not enough information to answer**
- If the sample size increased from 20 to 25, the margin of error for the confidence interval would (mark one answer):
☐ **increase** ☐ **decrease** ☐ **stay the same** ☐ **not enough information to answer**
- If the sample mean changed from 11.2 to 12.2 and the sample standard deviation stayed the same, the margin of error for the confidence interval would (mark one answer):
☐ **increase** ☐ **decrease** ☐ **stay the same** ☐ **not enough information to answer**

4. Carefully read the following questions and circle **one** answer for each of the following:

a. **True or False** If you choose to reduce the chance of making Type I error, you will also reduce the chance of making Type II error.

b. **True or False** If your decision is to Reject Ho, you could be making Type I error.

c. **True or False** It is better to collect and analyze the data before designing the experiment.

d. **True or False** If a hypothesis test is conducted at α=.01, and the experiment results in a p-value of 0.029, then the null hypothesis should be rejected.

e. **True or False** Testing for a difference between two population means should be conducted as a two-tailed test.

f. **True or False** A test with higher power has a lower chance of making type II error.

g. **True or False** If you want to reduce the chance of making Type I error , increase the value of α

h. **True or False** If the p-value of a test is 0.04, you can say with 96% confidence that Ha is true.

5. A Sports Medicine clinic was encouraging the use of a Conservative (non-surgical) protocol in the treatment of complete ruptures of the Achilles tendon. This protocol consists of a series of castings and TENS treatment followed by ultrasound and rigorous physical therapy. In a study performed by the Sports Medicine clinic, of the 1000 patients who received this treatment, 90 re-ruptured the tendon within two years of the original injury.
The Standard (surgical) protocol involves surgery to reattach the tendon followed by physical therapy. Of those who receive the Standard protocol, 13% re-rupture the tendon within two years of the original injury. Test the hypothesis that the Conservative protocol reduces this **proportion** of patients who re-rupture the tendon. (α=5%) Show all procedures of Hypothesis Testing.

6. A study claims adults spend 9 hours per weekend on chores. A researcher wanted to see if this claim was true by sampling 101 adults. The mean and standard deviation of time spent on chores per weekend is shown below:

$\overline{X} = 8.6 \qquad s = 2.2$

a. Test the alternative hypothesis (α=.05) that the mean time spent on chores per weekend is less than 9 hours. **SHOW ALL STEPS** using the p-value method.
b. Test the alternative hypothesis (α=.05) that population standard deviation is under 3. (critical value method)

7. A researcher is comparing a treatment population to past data a out average length of time to complete a paragraph (at least 10 minutes) and wants to test the null hypothesis: $H_O : \mu \geq 10$ at a significance level of 5%. The researcher is planning on independently sampling 29 students.
   a. What is the alternative hypothesis?
   b. What are the degrees of freedom for the t-distribution in the test statistic?
   c. Determine the Critical Value(s) and write the decision rule.

   The researcher has determined if $|\mu_0 - \mu_a| = 2$, the power of the test would be 60%. Determine the effect of making the following changes in the experiment and test (circle one answer for each):

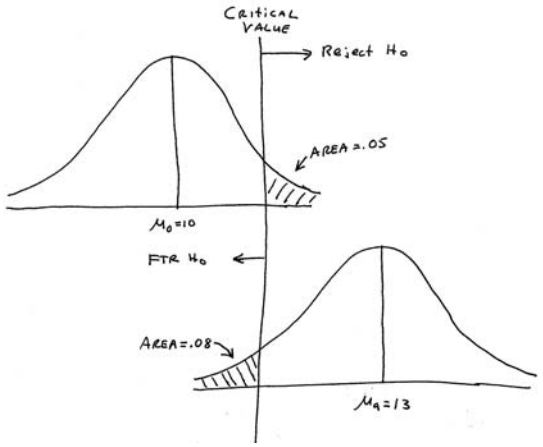
   a) Change the significance level from 5% to 1%: increase power reduce power no effect
   b) Change the sample size from 29 to 20: increase power reduce power no effect
   c) Instead of $|\mu_0 - \mu_a| = 2$, calculate power for $|\mu_0 - \mu_a| = 3$: increase power reduce power no effect

8. A stat student claims that over 60% of the students at De Anza oppose plus/minus grading. To prove this claim with 95% certainty, a survey was conducted and 140 out of 200 sampled students opposed plus/minus grading.

   a. State the Null and Alternative Hypotheses for this test.
   b. Determine α.
   c. Write the test statistic.
   d. Determine the Critical Value(s) and write the decision rule based (Critical Value Method).
   e. Make your decision and write a conclusion in non-statistical language.

9. The drawing below diagrams a hypothesis test for mean design under the Null Hypothesis (top drawing) and a specific Alternative Hypothesis (bottom drawing).

   a. State the Null and Alternative Hypotheses.
   b. What is the significance level of the test?
   c. What is the Power of the test when the population mean = 13?



10. This data is the starting monthly pay ($100's) for technicians in Santa Clara and Alameda counties (20 each).

Santa Clara	**32**	**29**	**35**	**34**	**38**	**35**	**33**	**28**	**26**	**27**	**28**	**29**	**30**	**31**	**37**	**40**	**20**	**31**	**36**	**30**
Alameda	**32**	**25**	**26**	**28**	**30**	**22**	**28**	**24**	**33**	**31**	**29**	**27**	**25**	**29**	**22**	**28**	**35**	**23**	**25**	**28**

For each the following questions, write the **Null and alternative hypothesis**, **degrees of freedom** (if applicable) and **choose the appropriate test statistic**. You do not need to conduct the tests.

1. One population, large sample Z test for mean 2. One population, small sample t test for mean 3. One population, Z test of proportion 4. One population, χ^2 test of standard deviation	5. Z-test: comparing two independent population means 6. t-test: independent samples, two population pooled variance. 7. t-test: independent samples, two population unequal variance. 8. t-test: dependent sampling, matched pairs

a. Test the hypothesis that Mean Salary for Santa Clara County technicians exceeds $3400
b. Test the hypothesis that the standard deviation for Alameda County technicians is $400
c. Test the hypothesis that more than 40% of the technicians (both counties combined) have a starting pay under $3000.
d. Test for a difference in starting pay between the 2 counties (you may assume equal variances, if necessary)

11. A company wants to increase employee contributions to its 401(k) savings plan. The savings rates are shown below for 15 employees before and after investment seminars presented by the management.

Employee Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Rate Before Seminar	0	0	0	2	3	3	5	5	5	6	8	10	10	10	15
Rate After Seminar	5	0	0	5	10	0	10	7	5	6	8	12	12	15	15

To test the hypothesis that the seminar increases the mean savings rate, state the appropriate hypothesis (Before is subscript 1), the degrees of freedom (if applicable) and then bubble the appropriate test statistic.

Ho_____

Ha_____

Degrees of Freedom _____
(if applicable)

- ○ One population, large sample Z test for mean
- ○ One population, small sample t test for mean
- ○ One population, Z test of proportion
- ○ One population, χ^2 test of standard deviation
- ○ Z-test: comparing two independent population means
- ○ t-test: independent samples, two population pooled variance.
- ○ t-test: independent samples, two population unequal variance.
- ○ t-test: dependent sampling, matched pairs

12. The head circumferences a random sample of two-month old boys and girls are shown below. The researcher wants to test the claim that boys and girls have different head sizes. Assume the significance level is 1% and you choose not to assume the populations variances are equal (which may possibly affect your test selection).

Boys	35.5	35.7	39.2	39.6	39.7	39.8	39.9	40.1	40.2	40.2	40.2	40.4	40.4	40.7
Girls	34.4	36.3	36.9	37.8	38.1	38.2	38.3	38.6	38.6	38.9	39.2	39.3	39.5	39.6

a. State the Null and Alternative Hypotheses for this test.
b. Determine α.
c. What is the test statistic? (Check one answer):
    A. t-test: mean vs. hypothesized value
    B. Z-test: comparing two independent population means
    C. t-test: independent samples, two population pooled variance.
    D. t-test: independent samples, two population unequal variance.
    E. t-test: dependent sampling, matched pairs
d. Write the decision rule using the p-value method.
e. The p-value is .038. Make your decision and write a conclusion in non-statistical language.

13. In problem 12, the sample variance for Boy's circumference is 2.737 and the sample variance for Girl's circumference is 2.039. Conduct an F test for equality of variances with a significance level of 5%. [Critical Values you might need: F(.05,13,13)=2.58 and F(.025,13,13)=3.12]

14. The sample standard deviation for the circumference of 14 newborn boy's heads is 1.65 centemeters. Find a 95% confidence interval for the population standard deviation boy's head circumferences.

15. (23pts) An industrial engineer compared the strength of two types of fasteners used in heavy machinery. The fasteners were put under extreme stress and the time in minutes until the fastener failed is recorded below along with the output of two possible t-tests. The method chosen was independent sampling for two populations. In order to determine whether to assume equal variances, the Hypothesis $H_O : \sigma_1^2 = \sigma_2^2$ was tested. The data is shown below along with the two-tail p-values for four different tests (not all of them needed). All tests will be conducted with $\alpha = .05$. Test for a difference in mean fastener strength between Method A and Method B.

Method A	Method B		Method A	Method B		
25	25		24.14	26.14	mean	
26	28		2.03	3.01	std. dev.	
26	25		14	14	n	
24	28					
23	29					
24	24					**pvalue**
22	28		Pooled variance t-test for comparing means			**0.0495**
26	24					
25	21		Unequal Variance t-test for comparing means			**0.0513**
20	30					
24	31		Matched pairs t-test for a difference of means			**0.1078**
21	24					
27	22		F test for comparing 2 population variances			**0.1705**

DESIGN 1: State your Hypothesis	**DATA:** Conduct the test and **circle** your decision (p-values for various tests are shown above) Reject Ho Fail to Reject Ho
DESIGN 2: State Significance Level and Decision Rule (p-value Method)	**CONCLUSION:** State your overall conclusion in language that is clear, relates to the original problem and is consistent with your decision.

Answers

Q1 a) 0.027 b) both A and B Decrease MOE

Q2 a) (31,7, 43.9) b) decrease c) decrease d) increase e) decrease

Q3 a) (9.23,13.17) b) increase c) not enough info d) decrease e) stay the same

Q4 a)F b) T c) F d) F e) T f) T g)F h)F

Q5 a: $Ho: p \geq .13$ $Ha: p < .13$ b:α=.05 c: Z-proportion d: Reject Ho if Z<-1.645 e: Z=-3.76 Reject Ho
The new Protocol is more effective.

Q6 a: $Ho: \mu \geq 9$ $Ha: \mu < 9$, α=.05 Model: t-test, 1 Pop df =100, so approx with Z, Results: pvalue=.0336, Reject Ho
Conclusion: Mean time spent on chores is less than 9 hours.

Q6 b: $Ho: \sigma \geq 3$ $Ha: \sigma < 3$, α=.05 Model: $\chi 2$ test of std dev, 1 Pop df =100, Reject Ho if $\chi 2$< 77.929
Results: $\chi 2$=53.24, Reject Ho Conclusion: Standard Deviation is under 3.

Q7 a: $Ha: \mu < 10$ b: df=28 c: Reject Ho if t<-1.701 d: reduce, reduce, increase

Q8 a: $Ho: p \leq .6$ $Ha: p > .6$ b:α=.05 c: Z-proportion d: Reject Ho if Z>1.645 e: Z=2.88 Reject Ho
More than 60% of students oppose +/- grading.

Q9 a: $Ho: \mu \leq 10$ $Ha: \mu > 10$ b: α=.05 c: Power=.92

Q10 a: $Ho: \mu \leq 3400$ $Ha: \mu > 3400$, df =19, test #2 b: $Ho: \sigma = 400$ $Ha: \sigma \neq 400$, df =19, test #4

Q10 c: $Ho: p \leq 0.4$ $Ha: p > 0.4$, df =N/A, test #3 d: $Ho: \mu 1 = \mu 2$ $Ha: \mu 1 \neq \mu 2$, df =38, test #6

Q11: $Ho: \mu_1 \geq \mu_2$ $Ha: \mu_1 < \mu_2$ df=14 matched pairs t-test

Q12: a: $Ho: \mu_1 = \mu_2$ $Ha: \mu_1 \neq \mu_2$ b:α=.01 c: t-test, ind, unequal variances d: Reject Ho pvalue<α
e: Fail to Reject Ho Insufficient Evidence to conclude boys and girls have different head sizes.

Q13: a: $Ho: \sigma_1^2 = \sigma_2^2$ $Ha: \sigma_1^2 \neq \sigma_2^2$ b:α=.05 c: $F = s_1^2 / s_2^2$ (because S_1 is larger) d: Reject Ho if F>3.12
e: F=1.34 Fail to Reject Ho Not enough evidence to claim variances are different.

Q14 (1.20,2.66) cm.

Q15:

DESIGN 1: State your Hypothesis

Ho $\mu_1 = \mu_2$

Ha $\mu_1 \neq \mu_2$

DATA: Conduct the test and **circle** your decision (p-values for various tests are shown above)

pvalue = .0495 <.05

(Reject Ho) Fail to Reject Ho

DESIGN 2: State Significance Level and Decision Rule (p-value Method)

$\alpha = .05$

Reject Ho if pvalue $< \alpha$

CONCLUSION: State your overall conclusion in language that is clear, relates to the original problem and is consistent with your decision.

There is a difference in mean flatness strength due to method.

DESIGN 3: Determine which of these 3 models you are using. (circle one answer). Justify your reasons for choosing the model

(a.) Pooled variance t-test

b. Unequal variance t-test

c. Matched pairs t-test

Based on pvalue for Ftest of Ho $\sigma_1 = \sigma_2$ Ha $\sigma_1 \neq \sigma_2$
pvalue = .1705 so
FTR Ho