Math 10 - Homework 3 Answers

1. Explain the difference between population parameters and sample statistics. What symbols do we use for the mean and standard deviation for each of these?
   Parameters are fixed values that are determined by the population. (μ, σ)
   Statistics are calculated from the sample and can change when different samples are taken (X-bar, s)

2. Consider the following probability distribution function of the random variable X which represents the number of bedrooms in a neighborhood's homes:

X	P(X)	xp(x)	(x-μ)	$(x-\mu)^2$	$(x-\mu)^2p(x)$
0	0.05	**0.0**	**-2.8**	**7.84**	**0.392**
1	0.1	**0.1**	**-1.8**	**3.24**	**0.324**
2	0.2	**0.4**	**-0.8**	**0.64**	**0.128**
3	0.4	**1.2**	**0.2**	**0.04**	**0.016**
4	0.15	**0.6**	**1.2**	**1.44**	**0.216**
5	**0.1**	**0.5**	**2.2**	**4.84**	**0.484**
		2.8			**1.56**

   a. Fill in the missing P(X)
      0.1
   b. Find the population mean of X.
      μ = 2.8 bedrooms
   c. Find the population variance and standard deviation of X.
      σ^2 = 1.56 σ = 1.249

3. 10% of all children at large urban elementary school district have been diagnosed with learning disabilities. 10 children are randomly and independently selected from this school district.

   a. Let X = the number of children with learning disabilities in the sample. What type of random variable is this?
      Binomial n=10, p=.10

   b. Find the mean and standard deviation of X.
      μ=(10)(.1)=1 σ^2=(10)(.1)(.9)=.9 σ = sqrt(.9) = .949

   c. Find the probability that exactly 2 of these selected children have a learning disability.
      from table: P(X=2)=.194

   d. Find the probability that at least 1 of these children has a learning disability.
      from table: P(X>=1) = 1-P(X=0) = 1-.349 = .651

   e. Find the probability that less than 3 of these children have a learning disability.
      from table: P(X<3) = .349+.387+.194 = .93

4. High Fructose Corn Syrup (HFCS) is a sweetener in food products that is linked to obesity and type II diabetes. The mean annual consumption in the United States in 2008 of HFCS was 60 lbs with a standard deviation of 20 lbs. Assume the population follows a Normal Distribution.

   a. Find the probability a randomly selected American consumes more than 50 lbs of HFCS per year.
      P(X>50) = P(Z>(50-60)/20)=P(Z>-0.50)=1-.3085=.6915

   b. Find the probability a randomly selected American consumes between 30 and 90 lbs of HFCS per year.
      P(30<X<90) = P((30-60)/20)<Z<(90-60)/20)=P(-1.50<Z<1.50)=.9332-.0668= .8664

   c. Find the 80^{th} percentile of annual consumption of HFCS.
      Z_{80}=0.84 X_{80}=60+(0.84)(20)=76.8 lbs. per year

   d. In a sample of 40 Americans how many would you expect consume more than 50 pounds of HFCS per year.
      P(X>50) = .6915 from part A. Expected Value = 40(.6915) = 27.7 or about 28 out of 40.

   e. Between what two numbers would you expect to contain 95% of Americans HFCS annual consumption?
      (60-1.96(20), 60+1.96(20)) or 20.8 to 99.2 lbs. per year

   f. Find the quartiles and Interquartile range for this population.
      Z_{25}=-0.67 Z_{50}=0 Z_{75}=0.67 X_{25}=60-(0.67)(20)=46.6 lbs X_{50}=60 lbs
      X_{75}=60+(0.67)(20)=73.4 lbs
      IQR=73.4-46.6= 26.8 lbs per year

   g. A teenager who loves soda consumes 105 lbs of HFCS per year. Is this result unusual? Use probability to justify your answer.
      P(X>105) = P(Z>(105-60)/20)=P(Z>2.25)=1 - 0.9878 = 0.0122 Unusual result

   h. In a sample of 16 Americans, what is the probability that the **sample mean** will exceed 57 pounds of HFCS per year?
      P(XBAR>57) = P(Z>(57-60)/(20/sqrt(16))=P(Z>-0.60)=1 - 0.2743 = 0.7257

   i. In a sample of 16 Americans, what is the probability that the **sample mean** will be between 50 and 70 pounds of HFCS per year.
      P(50<XBAR<70) = P((50-60)/(20/sqrt(16))<Z<(70-60)/(20/sqrt(16))
      =P(-2.00<Z<2.00)= = 0.9772 - 0.0228 = 0.9544

   j. In a sample of 16 Americans, between what two values would you expect to see 95% of the **sample means**?
      60 – 1.96*20/sqrt(16) , 60 + 1.96*20/sqrt(16)
      Between 50.2 and 69.8 lbs

5. A normally distributed population of package weights has a *mean* of 63.5 g and a *standard deviation* of 12.2 g.

   a. What percentage of this population weighs 66 g or more?
   P(X>66) = P(Z>(66-63.5)/12.2)=P(Z>0.20)=1-.5793=.4207

   b. What percentage of this population weighs 41 g or less?
   P(X<41) = P(Z<(41-63.5)/12.2)=P(Z<-1.84)=.0329

   c. What percentage of this population weighs between 41 g and 66 g?
   P(41<X<66) = P((41-63.5)/12.2)<Z<(66-63.5)/12.2)=P(-1.84<Z<0.20)=.5793-.0329=.5464

   d. Find the 60^{th} percentile for distribution of weights.
   Z_{60}=0.25 X_{60}=63.5+(0.25)(12.2)=66.55 g

   e. Find the three quartiles and the interquartile range.
   Z_{25}=-0.67 Z_{50}=0 Z_{75}=0.67 X_{25}=63.5-(0.67)(12.2)=55.3g X_{50}=63.5 X_{75}=63.5+(0.67)(12.2)=71.7g
   IQR=71.7-55.3=16.4 g

   f. If you sample 49 packages, find the probability the sample mean is over 66 g. Compare this answer to part a.
   P(Xbar>66) = P(Z>(66-63.5)/[12.2/sqrt(16)]=P(Z>0.82)=1-.7939=.2061 lower since sdev is lower.

   g. If you sample 49 packages, find the probability the sample mean is over 66 g. Compare this answer to part a.
   P(Xbar>66) = P(Z>(66-63.5)/[12.2/sqrt(49)]=P(Z>1.43)=1-.9236=.0764 lower since sdev is lower.

6. A pollster sampled 100 adults in California and asked a series of questions. The Central Limit Theorem for Proportions requires that np > 10 and n(1-p) > 10. Determine if these conditions are met for the following statements.

   a. 61% of Californians live in Southern California.
   np=61, n(1-p)=39 – both more than 10 - yes

   b. 92% of Californians support Deferred Action for Childhood Arrivals (DACA)
   np=92, n(1-p)=8 – n(1-p) less than 10 - no

   c. 8% of Californians have a felony conviction.
   np=8, n(1-p)=92 – np less than 10 - no

7. 24% of Californians have visited Yosemite National Park. A pollster samples 1000 Californians.

   a. Determine the expected value and standard deviation of the sample proportion.
   Mean = p = 0.24   std dev = sqrt(0.24(1-0.24)/1000) = 0.0135

   b. Determine that the condition for normality is satisfied.
   np=240, n(1-p)=760 – both more than 10 – yes

   c. Determine the probability the sample proportion exceeds 0.40.
   P(p-hat>0.40) = P(Z>(0.40 – 0.24)/0.0135) = P(Z>11.85) = 0