


Inferential Statistics and Probability a Holistic Approach


Chapter 12

One Factor Analysis of Variance (ANOVA)



This Course Material by Maurice Geraghty is licensed under a Creative Commons Attribution-ShareAlike 4.0 International License.
Conditions for use are shown here: <https://creativecommons.org/licenses/by-sa/4.0/>


1



Underlying Assumptions for ANOVA

- The F distribution is also used for testing the equality of more than two means using a technique called analysis of variance (ANOVA). ANOVA requires the following conditions:
 - The populations being sampled are normally distributed.
 - The populations have equal standard deviations.
 - The samples are randomly selected and are independent.

2



ANOVA Definitions


- **Factor** – categorical variable that defines the populations.
- **Response** – variable that is being measured.
- **Levels** – the number of choices for the factor, represented by k
- **Replicates** – the sample size for each level, n_1, n_2, \dots, n_k .
- If $n_1 = n_2 = \dots = n_k$, then the design is **balanced**.

- **H₀**: There is no difference in the mean <response in context> due to the <factor in context>.
- **H_a**: There is a difference in the mean <response in context> due to the <factor in context>.

3

Characteristics of F-Distribution

- There is a "family" of F Distributions.
- Each member of the family is determined by two parameters: the numerator degrees of freedom and the denominator degrees of freedom.
- F cannot be negative, and it is a continuous distribution.
- The F distribution is positively skewed.
- Its values range from 0 to ∞ .
 - As $F \rightarrow \infty$ the curve approaches the X-axis.



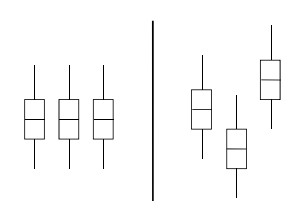
4

Analysis of Variance Procedure

- The Null Hypothesis:** the population means are the same.
- The Alternative Hypothesis:** at least one of the means is different.
- The Test Statistic:** $F = (\text{between sample variance}) / (\text{within sample variance})$.
- Decision rule:** For a given significance level α , reject the null hypothesis if $F(\text{computed})$ is greater than $F(\text{table})$ with numerator and denominator degrees of freedom.

5

ANOVA – Null Hypothesis



Ho is true - all means the same

Ho is false - not all means the same

6

ANOVA NOTES

- If there are k populations being sampled, then the df (numerator)=k-1
- If there are a total of n sample points, then df (denominator) = n-k
- The test statistic is computed by: $F = [(SS_F)/(k-1)] / [(SS_E)/(N-k)]$.
- SS_F represents the factor (between) sum of squares.
- SS_E represents the error (within) sum of squares.
- Let T_c represent the column totals, n_c represent the number of observations in each column, and ΣX represent the sum of all the observations.
- These calculations are tedious, so technology is used to generate the **ANOVA table**.

7

Formulas for ANOVA

$$SS_{Total} = \Sigma(X^2) - \frac{(\Sigma X)^2}{n}$$

$$SS_{Factor} = \Sigma\left(\frac{T_c^2}{n_c}\right) - \frac{(\Sigma X)^2}{n}$$

$$SS_{Error} = SS_{Total} - SS_{Factor}$$

8

ANOVA Table

Source	SS	df	MS	F
Factor	SS_{Factor}	k-1	SS_F/df_F	MS_F/MS_E
Error	SS_{Error}	n-k	SS_E/df_E	
Total	SS_{Total}	n-1		

9

EXAMPLE

Party Pizza specializes in meals for students. Hsieh Li, President, recently developed a new tofu pizza.

- Before making it a part of the regular menu she decides to test it in several of her restaurants. She would like to know if there is a difference in the mean number of tofu pizzas sold per day at the Cupertino, San Jose, and Santa Clara pizzerias for sample of five days.
- At the .05 significance level can Hsieh Li conclude that there is a difference in the mean number of tofu pizzas sold per day at the three pizzerias?

10

Example

	Cupertino	San Jose	Santa Clara	Total
	13	10	18	
	12	12	16	
	14	13	17	
	12	11	17	
			17	
T	51	46	85	182
n	4	4	5	13
Means	12.75	11.5	17	14
Σ^2	653	534	1447	2634

11


Example continued

$$SS_{Total} = 2634 - \frac{182^2}{13} = 86$$

$$SS_{Factor} = 2624.25 - \frac{182^2}{13} = 76.25$$

$$SS_{Error} = 86 - 76.25 = 9.75$$


12

 **Example 4** *continued*

ANOVA TABLE


Source	SS	df	MS	F
Factor	76.25	2	38.125	39.10
Error	9.75	10	0.975	
Total	86.00	12		

13

 **EXAMPLE 4** *continued*

- **Design:** $H_0: \mu_1 = \mu_2 = \mu_3$
 H_a : Not all the means are the same
- $\alpha = .05$
- Model: One Factor ANOVA
- H_0 is rejected if $F > 4.10$
- **Data:** Test statistic: $F = [76.25/2]/[9.75/10] = 39.1026$
- H_0 is rejected.
- **Conclusion:** There is a difference in the mean number of pizzas sold at each pizzeria.

14

 **One-way ANOVA: Cupertino, San Jose, Santa Clara**

Source	DF	SS	MS	F	P
Factor	2	76.250	38.125	39.10	0.000
Error	10	9.750	0.975		
Total	12	86.000			

S = 0.9874 R-Sq = 88.66% R-Sq(adj) = 86.40%

Individual 95% CIs For Mean Based on Pooled StDev

Level	N	Mean	StDev
Cupertino	4	12.750	0.957
San Jose	4	11.500	1.291
Santa Clara	5	17.000	0.707

12.0 14.0 16.0 18.0

15

Post Hoc Comparison Test

- Used for pairwise comparison
- Designed so the **overall** significance level is 5%.
- Use technology.
- Refer to **Tukey Test** Material in Supplemental Material.

16

Post Hoc Comparison Test

Grouping Information Using Tukey Method

	N	Mean	Grouping
Santa Clara	5	17.0000	A
Cupertino	4	12.7500	B
San Jose	4	11.5000	B

Means that do not share a letter are significantly different.

17

Post Hoc Comparison Test

Individual Value Plot of Cupertino, San Jose, Santa Clara

Group	Mean
Cupertino	12.75
San Jose	11.50
Santa Clara	17.00

18
