Inferential Statistics and Probability
a Holistic Approach

Chapter 11

Chi-square Tests for Categorical Data

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Characteristics of the Chi-Square Distribution

- The major characteristics of the chi-square distribution are:
  - It is positively skewed
  - It is non-negative
  - It is based on degrees of freedom
  - When the degrees of freedom change a new distribution is created

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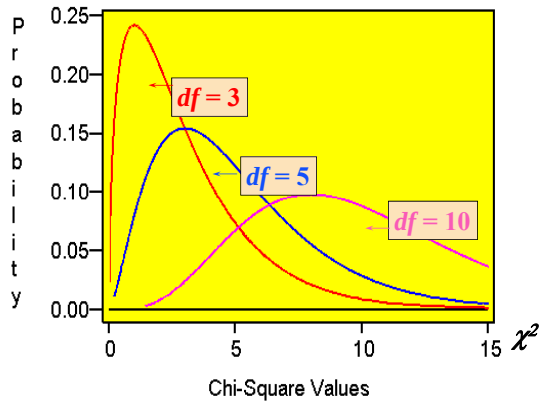

Goodness-of-Fit Test: Equal Expected Frequencies

- Let O_i and E_i be the observed and expected frequencies respectively for each category.
- H_0: there is no difference between Observed and Expected Frequencies
- H_a: there is a difference between Observed and Expected Frequencies
- The test statistic is: $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$
- The critical value is a chi-square value with (k-1) degrees of freedom, where k is the number of categories

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EXAMPLE 1

- The following data on absenteeism was collected from a manufacturing plant. At the .01 level of significance, test to determine whether there is a difference in the absence rate by day of the week.

Day	Frequency
Monday	95
Tuesday	65
Wednesday	60
Thursday	80
Friday	100

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EXAMPLE 1 *continued*

- Assume equal expected frequency: (95+65+60+80+100)/5=80

Day	O	E	(O-E)^2/E
Mon	95	80	2.8125
Tues	65	80	2.8125
Wed	60	80	5.0000
Thur	80	80	0.0000
Fri	100	80	5.0000
Total	**400**	**400**	**15.625**

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EXAMPLE 1 *continued*

- H_o: there is no difference between the observed and the expected frequencies of absences.
- H_a: there is a difference between the observed and the expected frequencies of absences.
- Test statistic: chi-square=$\Sigma(O-E)^2/E$=15.625
- Decision Rule: reject H_o if test statistic is greater than the critical value of 13.277. (4 df, α=.01)
- Conclusion: reject H_o and conclude that there is a difference between the observed and expected frequencies of absences.

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Goodness-of-Fit Test: Unequal Expected Frequencies

EXAMPLE 2

The U.S. Bureau of the Census (2000) indicated that 54.4% of the population is married, 6.6% widowed, 9.7% divorced (and not re-married), 2.2% separated, and 27.1% single (never been married).

A sample of 500 adults from the San Jose area showed that 270 were married, 22 widowed, 42 divorced, 10 separated, and 156 single.

At the .05 significance level can we conclude that the San Jose area is different from the U.S. as a whole?

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EXAMPLE 2 *continued*

Status	O	E	$\sum \frac{(O-E)^2}{E}$
Married	270	272	0.015
Widowed	22	33	3.667
Divorced	42	48.5	0.871
Separated	10	11	0.091
Single	156	135.5	3.101
Total	500	500	7.745

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EXAMPLE 2 *continued*

- **Design:** H_o: p_1=.544 p_2=.066 p_3=.097 p_4=.022 p_5=.271
  H_a: at least one p_i is different
- α=.05
- Model: Chi-Square Goodness of Fit, df=4
- H_o is rejected if $\chi^2 > 9.488$
- **Data:** $\chi^2 = 7.745$, Fail to Reject Ho
- **Conclusion:** Insufficient evidence to conclude San Jose is different than the US Average

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Contingency Table Analysis

- Contingency table analysis is used to test whether two traits or variables are related.
- Each observation is classified according to two variables.
- The usual hypothesis testing procedure is used.
- The *degrees of freedom* is equal to: (number of rows-1)(number of columns-1).
- The expected frequency is computed as: Expected Frequency = (row total)(column total)/grand total

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EXAMPLE 3

- In May 2014, Colorado became the first state to legalize the recreational use of marijuana.
- A poll of 1000 adults were classified by gender and their opinion about legalizing marijuana
- At the .05 level of significance, can we conclude that gender and the opinion about legalizing marijuana for recreational use are dependent events?

Marijuana should be	Men	Women	Total
Legal	270	230	500
Not Legal	205	245	450
Unsure	25	25	50
Total	500	500	1000

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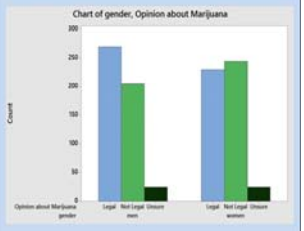
EXAMPLE 3 *continued*

Rows: Opinion about Marijuana
Columns: gender

1st value = Observed
2nd Value = Expected
3rd Value = Contribution to Chi-square

	men	women	All
Legal	270	230	500
	250	250	
	1.600	1.600	
Not Legal	205	245	450
	225	225	
	1.778	1.778	
Unsure	25	25	50
	25	25	
	0.000	0.000	
All	500	500	1000



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EXAMPLE 3 *continued*

- **Design:** H_o: Gender and Opinion are independent.
  H_a: Gender and Opinion are dependent.
- α=.05
- Model: Chi-Square Test for Independence, df=2
- H_o is rejected if $\chi^2 > 5.99$
- **Data:** $\chi^2 = 6.756$, Reject Ho
- **Conclusion:** Gender and opinion are dependent variables. Men are more likely to support legalizing marijuana for recreational use.

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