


## Inferential Statistics and Probability a Holistic Approach

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### Chapter 7 Central Limit Theorem



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## Distribution of Sample Mean

- Random Sample:  $X_1, X_2, X_3, \dots, X_n$ 
  - Each  $X_i$  is a Random Variable from the same population
  - All  $X_i$ 's are Mutually Independent
- $\bar{X}$  is a function of Random Variables, so  $\bar{X}$  is itself Random Variable.
- In other words, the Sample Mean can change if the values of the Random Sample change.
- What is the Probability Distribution of  $\bar{X}$  ?

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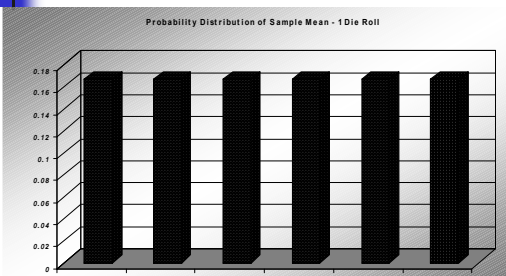
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## Example – Roll 1 Die

Probability Distribution of Sample Mean - 1 Die Roll



Die Roll	Probability
1	0.167
2	0.167
3	0.167
4	0.167
5	0.167
6	0.167

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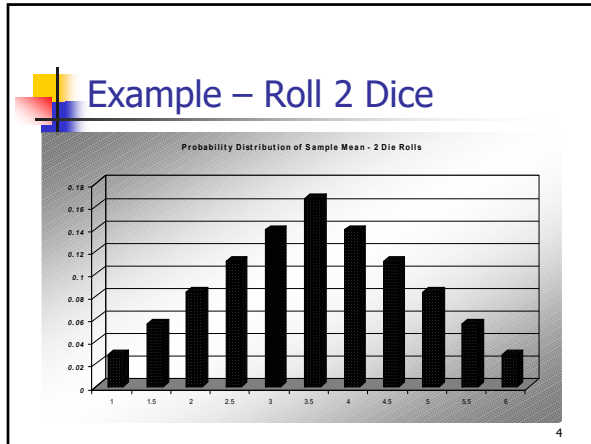
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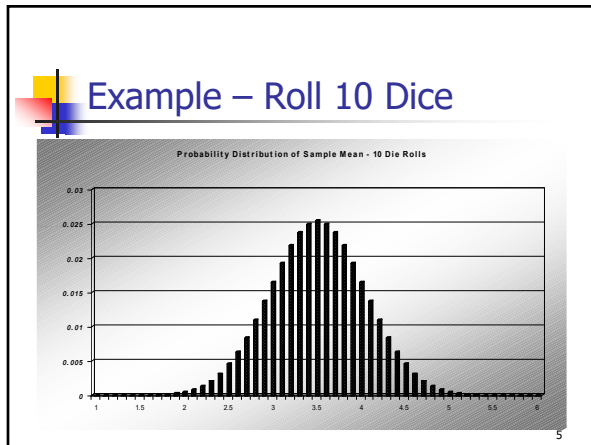
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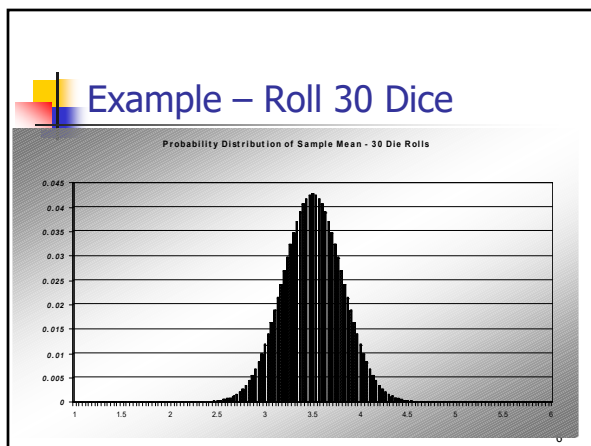
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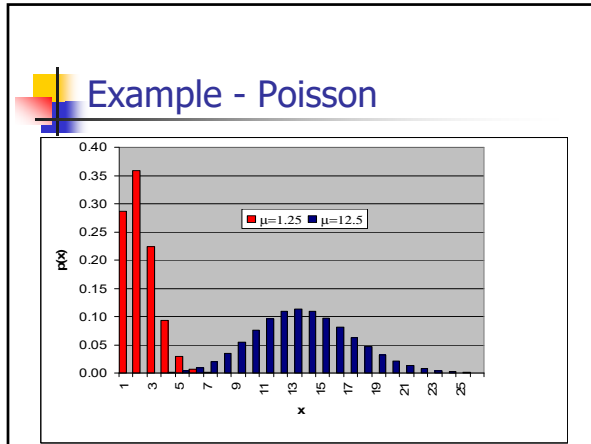
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### Central Limit Theorem – Part 1

- IF a Random Sample of **any size** is taken from a population with a **Normal Distribution** with mean=  $\mu$  and standard deviation =  $\sigma$

- THEN the distribution of the sample mean has a Normal Distribution with:
 
$$\mu_{\bar{X}} = \mu \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

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### Central Limit Theorem – Part 2

- IF a random sample of **sufficiently large size** is taken from a population with **any Distribution** with mean=  $\mu$  and standard deviation =  $\sigma$

- THEN the distribution of the sample mean has approximately a Normal Distribution with:
 
$$\mu_{\bar{X}} = \mu \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

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### Central Limit Theorem

3 important results for the distribution of  $\bar{X}$

- Mean Stays the same  

$$\mu_{\bar{X}} = \mu$$
- Standard Deviation Gets Smaller  

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$
- If n is sufficiently large,  $\bar{X}$  has a Normal Distribution

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### Example

The mean height of American men (ages 20-29) is  $\mu = 69.2$  inches. If a random sample of 60 men in this age group is selected, what is the probability the mean height for the sample is greater than 70 inches? Assume  $\sigma = 2.9$  inches.

$$P(\bar{X} > 70) = P\left(Z > \frac{(70 - 69.2)}{2.9/\sqrt{60}}\right)$$

$$= P(Z > 2.14) = 0.0162$$

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### Example (cont)

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### Example – Central Limit Theorem

The waiting time until receiving a text message follows an exponential distribution with an expected waiting time of 1.5 minutes. Find the probability that the mean waiting time for the 50 text messages exceeds 1.6 minutes.

$\mu = 1.5 \quad \sigma = 1.5 \quad n = 50$

Use Normal Distribution ( $n > 30$ )

$$P(\bar{X} > 1.6) = P\left(Z > \frac{(1.6 - 1.5)}{1.5/\sqrt{50}}\right) = P(Z > 0.47) = 0.3192$$

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### Central Limit Theorem Sample Proportion

- If X is a Random Variable from a Binomial Distribution with parameters n and p, an  $np > 10$  and  $n(1-p) > 10$ , then the following is true for the Sample Proportion,  $\hat{p}$  :

- $\mu_{\hat{p}} = p \quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$
- The Distribution of  $\hat{p}$  is approximately Normal.

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### Example

- 45% of all community college students in California receive fee waivers.
- Suppose you randomly sample 1000 community college students to determine the proportion of students with fee waivers in the sample.
- 483 of the sampled students are receiving fee waivers.
- Determine  $\hat{p}$  . Is the result unusual?

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