## Inferential Statistics and Probability
a Holistic Approach

Chapter 4
Probability



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## Probability

- Classical probability
  - Based on mathematical formulas
- Empirical probability
  - Based on the relative frequencies of historical data.
- Subjective probability
  - "one-shot" educated guess.

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## Examples of Probability

- What is the probability of rolling a four on a 6-sided die?
- What percentage of De Anza students live in Cupertino?
- What is the chance that the Golden State Warriors will be NBA champions in 2018?

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## Classical Probability

- Event
  - A result of an experiment
- Outcome
  - A result of the experiment that cannot be broken down into smaller events
- Sample Space
  - The set of all possible outcomes
- Probability Event Occurs
  - # of elements in Event / # Elements in Sample Space
- Example – flip two coins, find the probability of exactly 1 head.
  - {HH, HT, TH, TT}
  - P(1 head) = 2/4 = .5

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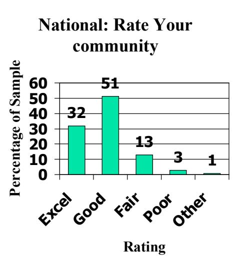
## Empirical Probability

- Historical Data
- Relative Frequencies
- Example: What is the chance someone rates their community as good or better?
- **0.51 + 0.32 = 0.83**

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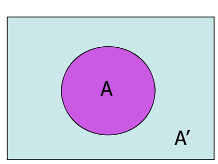
## Rule of Complement

- Complement of an event
- The event does not occur
- A′ is the complement of A
- P(A) + P(A′) =1
- P(A) = 1 – P(A′)

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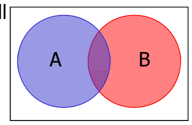
## Additive Rule

- The UNION of two events A and B is that either A or B occur (or both). (All colored parts)
- The INTERSECTION of two events A and B is that both A and B will occur. (Purple Part only)
- Additive Rule:
  P(A or B) = P(A) + P(B) – P(A and B)

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## Example

- In a group of students, 40% are taking Math, 20% are taking History.
- 10% of students are taking both Math and History.
- Find the Probability of a Student taking either Math or History or both.
- P(M or H) = 40% + 20% - 10% = 50%

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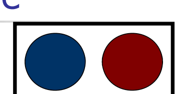
## Mutually Exclusive



- Mutually Exclusive
- Both cannot occur
- If A and B are mutually exclusive, then
  - P(A or B) = P(A) + P(B)
- Example roll a die
  - A: Roll 2 or less B: Roll 5 or more
  - P(A)=2/6 P(B)=2/6
  - P(A or B) = P(A) + P(B) = 4/6

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## Conditional Probability

- The probability of an event occuring GIVEN another event has already occurred.
- P(A|B) = P(A and B) / P(B)

- Example: Of all cell phone users in the US, 15% have a smart phone with AT&T. 25% of all cell phone users use AT&T. Given a selected cell phone user has AT&T, find the probability the user also has a smart phone.

- A=AT&T subscriber B=Smart Phone User
- P(A and B) = .15 P(A)=.25
- P(B|A) = .15/.25 = .60

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## Contingency Tables

- Two data items can be displayed in a contingency table.
- Example: auto accident during year and DUI of driver.

| | Accident | No Accident | Total |
|---|---|---|---|
| **DUI** | 70 | 130 | 200 |
| **Non- DUI** | 30 | 770 | 800 |
| **Total** | 100 | 900 | 1000 |

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## Contingency Tables

| | Accident | No Accident | Total |
|---|---|---|---|
| **DUI** | 70 | 130 | 200 |
| **Non- DUI** | 30 | 770 | 800 |
| **Total** | 100 | 900 | 1000 |

Given the Driver is DUI, find the Probability of an Accident.

A=Accident D=DUI

P(A and D) = .07 P(D) = .2

P(A|D) = .07/.2 = .35

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## Marginal, Joint and Conditional Probability

- **Marginal Probability** means the probability of a single event occurring.
- **Joint Probability** means the probability of the union or intersection of multiple events occurring.
- **Conditional Probability** means the probability of an event occurring given that another event has already occurred.

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## Creating Contingency Tables

- You can create a hypothetical contingency table from reported cross tabulated data.
- First choose a convenient sample size (called a radix) like 10000.
- Then apply the reported marginal probabilities to the radix of one of the variables.
- Then apply the reported conditional probabilities to the total values of one of the other variable.
- Complete the table with arithmetic.

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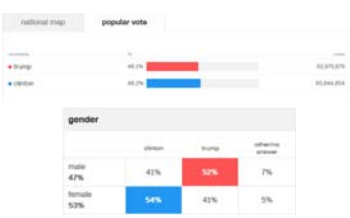
## Example

Create a two-way table from the cross tabulation of gender from the 2016 election results (from CNN)

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## Example

First select a radix (sample size) of 10000

| | GENDER | | |
|---|---|---|---|
| VOTED FOR | Female | Male | Total |
| Trump | | | |
| Clinton | | | |
| Other | | | |
| Total | | | 10000 |

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## Example

Then apply the marginal probabilities to the radix (53% female, 47% male)

| | GENDER | | |
|---|---|---|---|
| VOTED FOR | Female | Male | Total |
| Trump | | | |
| Clinton | | | |
| Other | | | |
| Total | 5300 | 4700 | 10000 |

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## Example

Then apply the cross tabulated percentages for each gender. Make sure the numbers add up.

| | GENDER | | |
|---|---|---|---|
| VOTED FOR | Female | Male | Total |
| Trump | 2173 | 2444 | |
| Clinton | 2862 | 1927 | |
| Other | 265 | 329 | |
| Total | 5300 | 4700 | 10000 |

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## Example

Finally, complete the table using arithmetic.

| | GENDER | | |
|---|---|---|---|
| VOTED FOR | Female | Male | Total |
| Trump | 2173 | 2444 | 4617 |
| Clinton | 2862 | 1927 | 4789 |
| Other | 265 | 329 | 594 |
| Total | 5300 | 4700 | 10000 |

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## Multiplicative Rule

- P(A and B) = P(A) x P(B|A)
- P(A and B) = P(B) x P(A|B)
- Example: A box contains 4 green balls and 3 red balls. Two balls are drawn. Find the probability of choosing two red balls.
- A=Red Ball on 1st draw   B=Red Ball on 2nd Draw
- P(A)=3/7   P(B|A)=2/6
- P(A and B) = (3/7)(2/6) = 1/7

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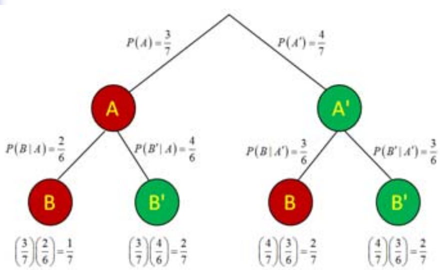
## Multiplicative Rule – Tree Diagram

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## Independence

- If A is not dependent on B, then they are INDEPENDENT events, and the following statements are true:
  - P(A|B)=P(A)
  - P(B|A)=P(B)
  - P(A and B) = P(A) x P(B)

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## Example

| | Accident | No Accident | Total |
|---|---|---|---|
| **DUI** | 70 | 130 | 200 |
| **Non- DUI** | 30 | 770 | 800 |
| **Total** | 100 | 900 | 1000 |

A: Accident D:DUI Driver

P(A) = .10 P(A|D) = .35 (70/200)

Therefore A and D are **DEPENDENT** events as P(A) < P(A|D)

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## Example

| | Accident | No Accident | Total |
|---|---|---|---|
| **Domestic Car** | 60 | 540 | 600 |
| **Import Car** | 40 | 360 | 400 |
| **Total** | 100 | 900 | 1000 |

A: Accident D:Domestic Car

P(A) = .10 P(A|D) = .10 (60/600)

Therefore A and D are **INDEPENDENT** events as P(A) = P(A|D)

Also P(A and D) = P(A)xP(D) = (.1)(.6) = .06

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## Random Sample

- A random sample is where each member of the population has an equally likely chance of being chosen, and each member of the sample is INDEPENDENT of all other sampled data.

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## Tree Diagram method

- Alternative Method of showing probability
- Example: Flip Three Coins
- Example: A Circuit has three switches. If at least two of the switches function, the Circuit will succeed. Each switch has a 10% failure rate if all are operating, and a 20% failure rate if one switch has already failed. Find the probability the circuit will succeed.

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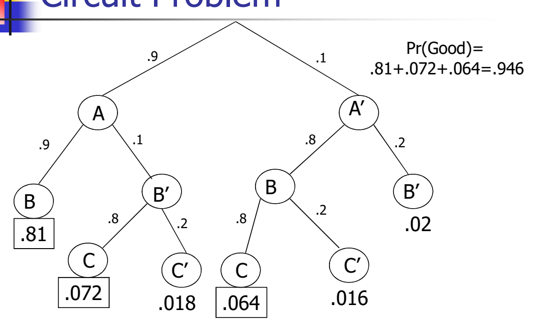
## Circuit Problem

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## Switching the Conditionality

- Often there are questions where you desire to change the conditionality from one variable to the other variable
- First construct a tree diagram.
- Second, create a Contingency Table using a convenient radix (sample size)
- From the Contingency table it is easy to calculate all conditional probabilities.

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## Example

- 10% of prisoners in a Canadian prison are HIV positive.
- A test will correctly detect HIV 95% of the time, but will incorrectly "detect" HIV in non-infected prisoners 15% of the time (false positive).
- If a randomly selected prisoner tests positive, find the probability the prisoner is HIV+

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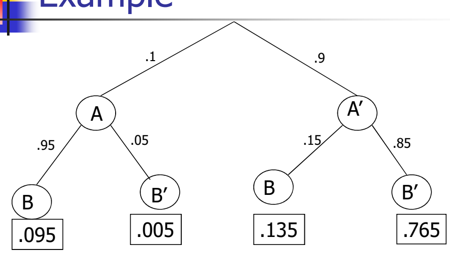
## Example




A=Prisoner is HIV+
B=Test is Positive for HIV

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## Example

| | HIV+ A | HIV- A' | Total |
|---|---|---|---|
| **Test+ B** | 950 | 1350 | 2300 |
| **Test- B'** | 50 | 7650 | 7700 |
| **Total** | 1000 | 9000 | 10000 |

$$P(A \mid B) = \frac{950}{2300} \approx .413$$

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